

The Case of the Missing Wormhole State^{*,**}

P.V. Moniz^{***}

Department of Applied Mathematics and Theoretical Physics
University of Cambridge
Silver Street, Cambridge, CB3 9EW, UK

ABSTRACT

The issue concerning the existence of wormhole states in locally supersymmetric minisuperspace models with matter is addressed. Wormhole states are apparently absent in models obtained from the more general theory of $N=1$ supergravity with supermatter. A Hartle-Hawking type solution can be found, even though some terms (which are scalar field dependent) cannot be determined in a satisfactory way. A possible cause is investigated here. As far as the wormhole situation is concerned, we argue here that the type of Lagrange multipliers and fermionic derivative ordering one uses may make a difference. A proposal is made for supersymmetric quantum wormholes to also be invested with a Hilbert space structure, associated with a maximal analytical extension of the corresponding minisuperspace.

Mystery stories seem to be a must in Britain. One just has to remember famous characters such as Sherlock Holmes, Hercule Poirot and Miss Marple and celebrated authors like Sir. A.C. Doyle and Agatha Christie. Furthermore, there is even a book entitled “Cambridge Colleges Ghosts” [0]. Hence, I hope that the title of this talk does not seem so strange after all. Let me then begin by some introductory remarks concerning our *mystery* case.

A quantum theory of gravity constitutes one of the foremost aspirations in theoretical physics [1]. The inclusion of supersymmetry could allow important achievements as well. Firstly, supersymmetry is an attractive concept with appealing possibilities in particle physics. The introduction of local supersymmetry and subsequently of supergravity provide an elegant gauge theory between bosons and fermions to which many hope nature has reserved a rightful place [2]. In fact, $N=1$ supergravity is a (Dirac) square root of gravity [3]: physical states in the quantum theory must satisfy the supersymmetry constraints

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*** e-mail address: prlv10@amtp.cam.ac.uk

which then imply with the quantum algebra that the Hamiltonian constraints also to be satisfied [3,4,5]. Secondly, ultraviolet divergences could be removed by the presence of the extra symmetry [6]. Thirdly, it was suggested [7] that Planckian effective masses induced by wormholes could be eliminated with supersymmetry.

Quite recently, an important result was achieved [8]. Namely, addressing the question of why the existence of a Hartle-Hawking [9] solution for Bianchi class A models in pure N=1 supergravity [10-14] seemed to depend on the homogeneity condition for the gravitino [12]. In fact, it does not and it is now possible to find a Hartle-Hawking and wormhole [15] solutions in the same spectrum [8,43]. This result requires the inclusion of all allowed gravitational degrees of freedom into the Lorentz invariant fermionic sectors of the wave function. However, there are many other issues in supersymmetric quantum gravity which remain unsolved. On the one hand, why no physical states are found when a cosmological constant is added [16-18] (nevertheless, a Hartle-Hawking solution was obtained for a $k = 1$ FRW model) [Extending the framework presented in ref. [8] and using Ashtekar variables, it was shown in ref. [44] that the exponential of the Chern-Simons functional constitute one case of solutions] and on the other hand, why the minisuperspace solutions have no counterpart in the full theory because states with zero (bosonic) or a finite number of fermions are not possible there [19]. A possible answer to the latter could be provided within the framework presented in ref. [8]. But another problem has also been kept without an adequate explanation: the apparent absence of wormhole states either in some FRW [20,21] or Bianchi IX models [22] when supermatter is included¹. In addition, a Hartle-Hawking type solution can be found, even though some terms (which are scalar field dependent) cannot be determined in a satisfactory way.

Classically, wormholes join different asymptotic regions of a Riemannian geometry. Such solutions can only be found when certain types of matter fields are present [15].

¹ Other interesting issues in supersymmetric quantum gravity/cosmology are: a) obtaining conserved currents in minisuperspace from the wave function of the universe, Ψ [23]; b) obtaining physical states in the full theory (are there any? how do they look?) and possibly checking the conjecture made in [8]; c) why there are no physical states in a locally supersymmetric FRW model with gauged supermatter [24] but one can find them in a locally supersymmetric FRW model with Yang-Mills fields [25].

However, it seems more natural to study quantum wormhole states, i.e., solutions of the Wheeler-DeWitt equation [15,26-29]. It is thought that wormholes may produce shifts in effective masses and interaction parameters [30,31]. Moreover, wormholes may play an important role which could force the cosmological constant to be zero [32]. The wormhole ground state may be defined by a path integral over all possible asymptotic Euclidian 4-geometries and matter fields whose energy-momentum tensor vanishes at infinity. Excited wormhole states would have sources at infinity. However, the question concerning the main differences between a wormhole ground state and the excited states does not bear a simple answer. In fact, if one has found the ground state (like in [15,36]) then excited states may be obtained from the repeated application of operators (like $\frac{\partial}{\partial\phi}$, e.g.) and implementing their orthonormality. But it is another issue if one happens to find a set of solutions from the Wheeler-DeWitt equation and tries to identify which correspond to a wormhole ground state or to excited states. Recent investigations on this problem [26,28] claim that what may be really relevant is to use the whole basis of wormhole solutions (namely, to calculate the effects of wormhole physics from Green's functions, where these have been factorized by introducing a *complete* set of wormhole states [15]) and not just trying to identify and label an explicit expression which would correspond either to a wormhole ground state or an excited one.

The Hartle-Hawking (or no-boundary proposal) [1,9] solution is expressed in terms of a Euclidian path integral. It is essentially a topological statement about the class of histories summed over. To calculate the no-boundary wave function we are required to regard a three-surface as the *only* boundary of a compact four-manifold, on which the four-metric is $g_{\mu\nu}$ and induces h_{ij}^0 on the boundary, and the matter field is ϕ and matches ϕ_0 on the boundary as well. We are then instructed to perform a path integral over all such $g_{\mu\nu}$ and ϕ within all such manifolds. For manifolds of the form of $\mathbf{R} \times \Sigma$, the no-boundary proposal indicates us to choose initial conditions at the initial point as to ensure the closure of the four geometry. It basically consists in setting the initial three-surface volume $h^{1/2}$ to zero but also involve regular conditions on the derivatives of the remaining components of the three-metric and the matter fields [1,9].

Let me briefly exemplify how wormhole states seem to be absent and why a Hartle-

Hawking solution is only partially determined. Considering the more general theory of N=1 supergravity with supermatter [33], one takes a $k = +1$ FRW model with complex scalar fields $\phi, \bar{\phi}$, their fermionic partners, $\chi_A, \bar{\chi}_{A'}$, and a two-dimensional spherically symmetric Kähler geometry. The main results were shown **not** to depend on the fermionic derivative factor ordering and possible Kähler geometry [21]. Using the homogeneous FRW Ansatz for the fields (which for the gravitino is $\psi_i^A = e_i^{AA'} \bar{\psi}_{A'}$ [35,36]), redefining $\chi_A \rightarrow a^{\frac{3}{2}}(1 + \phi\bar{\phi})^{-1}\chi_A$, $\psi_A \rightarrow a^{\frac{3}{2}}\psi_A$ to get simple Dirac brackets and using instead $\bar{\psi}_A = 2n_A^{B'}\bar{\psi}_{B'}$, $\bar{\chi}_A = 2n_A^{B'}\bar{\chi}_{B'}$ the supersymmetry constraints are

$$S_A = \frac{1}{\sqrt{2}}(1 + \phi\bar{\phi})\chi_A\pi_\phi - \frac{i}{2\sqrt{6}}a\pi_a\psi_A - \sqrt{\frac{3}{2}}\sigma^2 a^2\psi_A - \frac{5i}{4\sqrt{2}}\bar{\phi}\chi_A\bar{\chi}_B\chi^B \\ + \frac{1}{8\sqrt{6}}\psi_B\bar{\psi}_A\psi^B - \frac{i}{4\sqrt{2}}\bar{\phi}\chi_A\psi^B\bar{\psi}_B + \frac{5}{4\sqrt{6}}\chi_A\psi^B\bar{\chi}_B + \frac{\sqrt{3}}{4\sqrt{2}}\chi^B\bar{\chi}_A\psi_B - \frac{1}{2\sqrt{6}}\psi_A\chi^B\bar{\chi}_B \quad (1)$$

and its hermitian conjugate. Note that these expressions were obtained directly from a canonical action of the form $\int dt(p\dot{q} - H)$, where $H = \mathcal{N}\mathcal{H} + \psi_0^A S_A + \psi_0^{A'} \bar{S}_{A'}$. \mathcal{N} is the lapse function. Here, one uses $\hbar = 1$ and $\sigma^2 = 2\pi^2$. We choose $(\chi_A, \psi_A, a, \phi, \bar{\phi})$ to be the coordinates and $(\bar{\chi}_A, \bar{\psi}_A, \pi_a, \pi_\phi, \pi_{\bar{\phi}})$ to be the momentum operators.

Some criteria have been presented to determine a suitable factor ordering. This problem is related to the presence of cubic terms in the supersymmetry constraints. Basically, $S_A, \bar{S}_A, \mathcal{H}$ could be chosen by requiring that [35,37]:

1. $S_A\Psi = 0$ describes the transformation properties of Ψ under right handed supersymmetry transformations (in the (a, ψ_A) representation),
2. $\bar{S}_A\Psi = 0$ describes the transformation properties of Ψ under left handed supersymmetry transformations (in the $(a, \bar{\psi}_A)$ representation),
3. S_A, \bar{S}_A are Hermitian adjoints with respect to an adequate inner product [5],
4. A Hermitian Hamiltonian \mathcal{H} is defined by consistency of the quantum algebra.

However, not all of these criteria can be satisfied simultaneously (cf. [35,37]). An arbitrary choice is to satisfy **1,2,4** as in here and [20,21,35,37,38]. Another possibility (as in [20,21,36]) is to go beyond this factor ordering and insist that S_A, \bar{S}_A could still be related by a Hermitian adjoint operation (requirement **3.**). If one adopts this then there

are some quantum corrections to S_A, \bar{S}_A (namely, adding terms linear in ψ_A, χ_A to S_A and linear in $\bar{\psi}_A, \bar{\chi}_A$ to \bar{S}_A) which nevertheless modify the transformation rules for the wave function under supersymmetry requirements **1,2**.

Following the ordering used in ref.[20,21,35,37,38], one puts all the fermionic derivatives in S_A on the right. In \bar{S}_A all the fermionic derivatives are on the left.

The Lorentz constraint $J_{AB} = \psi_{(A}\bar{\psi}_{B)} - \chi_{(A}\bar{\chi}_{B)}$ imply for Ψ

$$\Psi = A + iB\psi^C\psi_C + C\psi^C\chi_C + iD\chi^C\chi_C + E\psi^C\psi_C\chi^D\chi_D, \quad (2)$$

where A, B, C, D , and E are functions of a, ϕ and $\bar{\phi}$ only. Using eq. (10) and its hermitian conjugate, one gets four equations from $S_A\Psi = 0$ and another four equations from $\bar{S}_A\Psi = 0$ (all first order differential equations!) which give

$$A = f(\bar{\phi})\exp(-3\sigma^2 a^2), \quad E = g(\phi)\exp(3\sigma^2 a^2) \quad (3)$$

where f, g are arbitrary anti-holomorphic and holomorphic functions of ϕ , respectively. Decoupling the equations for B, C, D (cf. ref. [21] for more details) one finds

$$B = h(\bar{\phi})(1 + \phi\bar{\phi})^{-\frac{1}{2}}a^3\exp(3\sigma^2 a^2), \quad C = 0, \quad D = k(\phi)(1 + \phi\bar{\phi})^{-\frac{1}{2}}a^3\exp(-3\sigma^2 a^2). \quad (4)$$

The result (4) is direct consequence that one could not find a consistent (Wheeler-DeWitt type) second-order differential equation for C and hence to B, D . It came directly from the corresponding first order differential equations. Changing S_A, \bar{S}_A in order that they can be related by some Hermitian adjoint transformation (**3.**) gives essentially the same outcome [21]. With a two-dimensional flat Kähler geometry one gets a similar result.

While Lorentz invariance allows the pair $\psi_A\chi^A$ in (2), supersymmetry rejects it. A possible interpretation could be that supersymmetry transformations forbid any fermionic bound state $\psi_A\chi^A$ by treating the spin- $\frac{1}{2}$ fields ψ^A, χ^B differently.

A Hartle-Hawking wave function² could be identified in the fermionic filled sector, say, $g(\phi)\exp(3\sigma^2 a^2)$, but for particular expressions of $g(\phi)$. We notice though that the Lorentz

² The Hartle-Hawking solution could not be found in the Bianchi-IX model of ref. [22]. Either a different homogeneity condition (as in [12]) for ψ_i^A or the framework of [8] could assist us in this particular problem.

and supersymmetry constraints are not enough to specify $g(\phi)$. A similar situation is also present in ref. [36], although an extra multiplicative factor of a^5 multiplying $g(\phi)$ induces a less clear situation. In fact, no attempt was made in ref. [36,38] to obtain a Hartle-Hawking wave function solution. Being $N = 1$ supergravity considered as a square root of general relativity [3], we would expect to be able to find solutions of the type $e^{ik\phi}e^{a^2}$. These would correspond to a FRW model with a massless minimally coupled scalar field in ordinary quantum cosmology [1,41].

In principle, there are no physical arguments for wormhole states to be absent in $N=1$ supergravity with supermatter. In ordinary FRW quantum cosmology with scalar matter fields, the wormhole ground state solution would have a form like $e^{-a^2 \cosh(\rho)}$, where ρ stands for a matter fields function [15,26-28]. However, such behaviour is not provided by eqs. (3), (4). Actually, it seems quite different. Moreover, we may ask in which conditions can these solutions be accommodated in order for wormhole type solutions to be obtained. The arbitrary functions $f(\phi, \bar{\phi}), g(\phi, \bar{\phi}), h(\phi, \bar{\phi}), k(\phi, \bar{\phi})$ do not allow to conclude unequivocally that in these fermionic sectors the corresponding bosonic amplitudes would be damped at large 3-geometries for any allowed value of $\phi, \bar{\phi}$ at infinity. Claims were then made in ref. [20,21] that no wormhole states could be found. The reasons were that the Lorentz and supersymmetry constraints do not seem sufficient in this case to specify the $\phi\bar{\phi}$ dependence of f, g, h, k .

Hence, one has a canonical formulation of $N=1$ supergravity which constitutes a (Dirac) like square root of gravity [3,4,5]. Quantum wormhole and Hartle-Hawking solutions were found in minisuperspaces for pure $N=1$ supergravity [8,10-14,17,18,34-35,37] but the former state is absent in the literature ³, for pure gravity cases [1,9,15,26-28]. Hartle-Hawking wave functions and wormhole ground states are present in ordinary minisuperspace with matter [1,9,15,26-28]. When supersymmetry is introduced [20-22,35-38] one faces some problems within the more general theory of $N=1$ supergravity with supermatter [33] (cf. ref. [20-22]) as far as Hartle-Hawking or wormhole type solutions are

³ Notice that for pure gravity neither classical or quantum wormhole solutions have been produced in the literature. A matter field seems to be required: the “throat” size is proportional to $\sqrt{\kappa}$ where κ represents the (conserved) flux of matter fields.

concerned. An attempt [38] using the constraints present in [35,37] but the ordering employed above, also seemed to have failed in getting wormhole states. In addition, a model combining a conformal scalar field with spin- $\frac{1}{2}$ fields (expanded in spin- $\frac{1}{2}$ hyperspherical harmonics and integrating over the spatial coordinates [30]) did not produce any wormhole solution as well [39]. However, ref. [36] clearly represents an opposite point of view, as it explicitly depicts wormhole ground states in a locally supersymmetric setting.

It might be interesting to point that the constraints employed in [36] (and also in [35,37,38]) were derived from a *particular* model constructed in [40], while ours [21] come directly from the *more general* theory of N=1 supergravity coupled to supermatter [33]. Moreover, there are many differences between the expressions in [34=37] and the one hereby (see also [21]), namely on numerical coefficients.

Let me sketch briefly how the supersymmetry constraints expressions in [36] were obtained. First, at the pure N=1 supergravity level, the following re-definition of fermionic non-dynamical variables

$$\rho^A \sim a^{-1/2} \psi_0^A + \mathcal{N} a^{-2} n^{AA'} \overline{\psi}_{A'}, \quad (5)$$

and its hermitian conjugate were introduced for a FRW model, changing the supersymmetry and Hamiltonian constraints. As a consequence, no fermionic terms were present in $\mathcal{H} \sim \{S_A, \overline{S}_A\}$ and no cubic fermionic terms in the supersymmetry constraints. Hence, no ordering problems with regard to fermionic derivatives were present. The model with matter was then extracted *post-hoc* [35,37] from a few basic assumptions about their general form and supersymmetric algebra. This simplified route seemed to give similar expressions, up to minor field redefinitions, to what one would obtain for a reduced model from the *particular* theory presented in [40], as stated in [35,37]. Note that cubic fermionic terms like $\psi \overline{\psi} \psi$ or $\psi \overline{\chi} \chi$ are now present but the former is absent in the pure case. In ref. [35,37,38], criteria **1,2,4** were used for the fermionic derivative ordering, while in ref. [36] one insisted to accomodate an Hermitian adjoint relation between the supersymmetry transformations (**3.**). It so happens that a wormhole ground state was found in the former but not in the latter. In ref. [20,21] the same possibilities for using these criteria were employed but with supersymmetry and Hamiltonian constraints directly obtained from $\psi_0^A, \overline{\psi}_0^{A'}, \mathcal{N}$ (see eq.

(5)). Apparently, no wormhole states were present. Moreover, we also recover a solution which satisfied only partially the no-boundary proposal conditions (see eq. (3)). A similar but yet less clear situation also seems to be present in ref. [36].

The issue concerning the existence or not of wormhole and Hartle-Hawking quantum cosmological states for minisuperspaces within $N=1$ supergravity with supermatte is therefore of relevance [42]. The current literature on the subject is far from a consensus. No explanation has been provided for the (apparent) opposite conclusions [20,21,33] concerning the existence of wormhole states and to point out which is right and why. Furthermore, it does not seem possible for the procedure presented in [8] to solve this conundrum.

Here an answer for this particular problem is presented. The explanation is that choosing the type of Lagrange multipliers and the fermionic derivative ordering one uses makes a difference. Our arguments are as follows.

On the one hand, the quantum formulation of wormholes in ordinary quantum cosmology has been shown to depend on the lapse function [27,28]. Such ambiguity has already been pointed out in [41] (see also [45]) but for generic quantum cosmology and related to bosonic factor ordering questions in the Wheeler-DeWitt operator. An ordering is necessary in order to make predictions. A proposal was made that the kinetic terms in the Wheeler-DeWitt operator should be the Laplacian in the natural (mini)superspace element of line, i.e., such that it would be invariant under changes of coordinates in minisuperspace [41]. Basically, this includes the Wheeler-DeWitt operator to be locally self-adjoint in the natural measure generated by the above mentioned element of line. However, it suffers from the problem that the connection defined by a minisuperspace line element like $ds^2 = \frac{1}{N} f_{\mu\nu} dq^\mu dq^\nu$ could not be linear on \mathcal{N} . This would then lead to a Wheeler-DeWitt operator *not* linear in \mathcal{N} as it would be in order that \mathcal{N} be interpreted as a Lagrange multiplier (it was also proposed in ref. [41] that this possible non-linearity dependence on \mathcal{N} could cancel out in theories like supergravity where bosons and fermions would be in equal number of degrees of freedom). For each choice of \mathcal{N} , there is a different metric in minisuperspace, all these metrics being related by a conformal transformation [46]. Therefore, for each of these choices, the quantization process will be different. In fact, for a minisuperspace consisting of a FRW geometry and homogeneous scalar field, a con-

formal coupling allows a more general class of solutions of the Wheeler-DeWitt equation than does the minimally coupled case, even if a one-to-one correspondence exists between bound states [46].

For some choices of \mathcal{N} the quantization are even inadmissible, e.g, when $\mathcal{N} \rightarrow 0$ too fast for vanishing 3-geometries in the wormhole case. Basically, requiring regularity for Ψ at $a \rightarrow 0$ is equivalent to self-adjointness for the Wheeler-DeWitt operator at that point. Such extension would be expected since wormhole wave functions calculated via a path integral are regular there. Three-geometries with zero-volume would be a consequence of the slicing procedure which has been carried. In other words, $a = 0$ simply represents a coordinate singularity in minisuperspace. An extension for (and beyond it), similar to the case of the Rindler wedge and the full Minkowski space, would be desirable. The requirement that the Wheeler-DeWitt operator be self-adjoint selects a scalar product and a measure in minisuperspace. Gauge choices of \mathcal{N} that vanish too fast when $a \rightarrow 0$ will lead to problems as the minisuperspace measure will be infinite at (regular) configurations associated with vanishing three-geometries volume. The difference on the quantization manifests itself in the Hilbert space structure of the wormhole solutions due to the scalar product dependence on \mathcal{N} and not in the structure of the Wheeler-DeWitt operator or path integral. More precisely, the formulation of global laws, i.e., finding boundary conditions for the Wheeler-DeWitt equation in the wormhole case, equivalent to the ones in the path integral approach, could depend on the choice of \mathcal{N} but not the local laws in minisuperspace³.

On the other hand, a similar effect seems to occur when local supersymmetry transformations are present. Besides the lapse function, we have now the time components of the gravitino field, ψ_0^A , and of the torsion-free connection ω_{AB}^0 as Lagrange multipliers. If one uses transformation (5) but without the last term, then the supersymmetry and Hamiltonian constraints read (in the pure case):

$$S_A = \psi_A \pi_a - 6ia\psi_A + \frac{i}{2a} n_A^{E'} \psi^E \psi_E \bar{\psi}_{E'}, \quad (6a)$$

³ Physical results such as effective interactions are independent of the choice of \mathcal{N} due to the way the corresponding path integrals are formulated.

$$\bar{S}_{A'} = \bar{\psi}_{A'} \pi_a + 6ia \bar{\psi}_{A'} - \frac{i}{2a} n_E^{A'} \bar{\psi}^{E'} \psi_E \bar{\psi}_{E'}, \quad (6b)$$

$$\mathcal{H} = -a^{-1}(\pi_a^2 + 36a^2) + 12a^{-1} n^{AA'} \psi_A \bar{\psi}_{A'}. \quad (6c)$$

If $\rho_A, \bar{\rho}_{A'}$ had been used instead of $\psi_0^A, \bar{\psi}_0^{A'}$ then the second terms in (6a)-(6c) would be absent. I.e., for the transformation (5) the corresponding supersymmetry constraints and the Hamiltonian are either linear or free of fermionic terms (cf. eq. (1) and ref. [34,35,37] as well). What seems to have been gone unnoticed is the following. *Exact* solutions of $S_A \Psi = 0$ and $\bar{S}_A \Psi = 0$ (using the criteria **1,2,4**) in the pure case for (6a),(6b) with or without second term are $A_1 = e^{-3a^2}$ and $A_2 = e^{3a^2}$, respectively, for $\Psi = cA_1 + dA_2 \psi_A \bar{\psi}^A$ where c, d are constants. This Ψ represents a linear combination of of WKB solutions of $\mathcal{H}\Psi = 0$, obtained from the corresponding Hamilton-Jacobi equation, i.e., they represent a *semi-classical approximation*, *but only* for the \mathcal{H} without the second term in (6c), i.e., when (5) is fully employed. Strangely it does not for the full expression in (6c); in fact the function e^{3a^2} would have to be replaced.

Hence the choice between ρ_A and ψ_0^A directly affects any consistency between the quantum solutions of the constraints (6a)-(6c). Moreover, an important point (which will be stressed later) is that the Dirac-like equations in ref. [36] lead consistently to a set of Wheeler-DeWitt equations (like in [35,37,38]) but that could not be entirely achieved in ref. [20,21]. As explained in eq. (4), the difficulty in determining the $\phi, \bar{\phi}$ dependence of f, g, h, k (and therefore to access on the existence of wormhole states) is related to the fact that $C = 0$, which is an indication as well that corresponding Wheeler-DeWitt equations could not be obtained from the supersymmetry constraints.

Choosing (5) one achieves the simplest form for the supersymmetry and Hamiltonian constraints and their Dirac brackets. This is important at the pure case level, as far as the solutions of $S_A \Psi = 0$ and $\bar{S}_A \Psi = 0$ are concerned. Moreover, fermionic factor ordering become absent in that case. If one tries to preserve this property through a *post-hoc* approach [35,37] when going to the matter case (keeping a simplified form for the constraints and algebra) then one might hope to avoid any problems like the ones referred

to in eq. (4). In addition, using the fermionic ordering of [36] where one accomodates the Hermitain adjointness with **1,2,4** up to minor changes relatively to **1,2**, one does get a wormhole ground state. Thus, there seems to be a relation between a choice of Lagrange multipliers (which simplifies the constraints and the algebra in the pure case), fermionic factor ordering (which may become absent in the pure case) and obtaining from the supersymmetry constraints second order consistency equations (i.e., Wheeler-DeWitt type equations). The failure of this last one is the reason why $C = 0$ and f, g, h, k cannot be determined from the algebra. Different choices of ψ_0^A or ρ_A , then of fermionic derivative ordering will lead to different supersymmetry constraints and to different solutions for the quantization of the problem. It should also be stressed that from the supersymmetric algebra a combination of two supersymmetry transformations, generated by S_A and $\bar{S}_{A'}$ and whose amount is represented by the Lagrange multipliers $\psi_0^A, \bar{\psi}_0^{A'}$, will be (essentially) equivalent to a transformation generated by the Hamiltonian constraint and where the lapse function is the corresponding Lagrange multiplier.

So, how should the search for wormholes ground states⁵ in N=1 supergravity be approached? One possibility would be to employ a transformation like (25) (see [35]). In fact, using it from the begining in our case model it will change some coefficients in the supersymmetry constraints as it can be confirmed. As a consequence, we are then allowed to get consistent second order differential equations from $S_A \Psi = 0$ and $\bar{S}_{A'} \Psi = 0$. Hence, a line equivalent to the one followed in ref. [36] can be used and a wormhole ground state be found. Alternatively, we could restrict to the *post-hoc* approach introduced and followed throughout in [34,38] as explained above. Another possibility, is to extend the approach introduced by L. Garay [26-29] in ordinary quantum cosmology to the cases where local supersymmetry is present. The basic idea is that what is really relevant is to determine a *whole* basis of wormhole solutions of the associated Wheeler-DeWitt operators, not just trying to identify one single solution like the ground state from a all set of solutions. Hence,

⁵ Regarding the Hartle-Hawking solutions it seems it can be obtained straightforwardly either up to a specific definition of homogeneity [12] or following the approach in [8]. This might help in regarding the results found in [22] with respect to the Hartle-Hawking solution.

one ought to adequately define what a basis of wormhole solutions means. In this case, we could be able to still use any Lagrange multiplier (just as ψ_0^A), avoiding having to find a redefinition of fermionic variables as in (5) but for the matter case in question (scalar, vector field, etc).

Basically, improved boundary conditions for wormholes can be formulated by requiring square integrability in the *maximally extended* minisuperspace [27,28]. This condition ensures that Ψ vanishes at the *truly* singular configurations and guarantees its regularity at any other (coordinate) one, including vanishing 3-geometries. A maximally extended minisuperspace and a proper definition of its boundaries in order to comply with the behaviour of Ψ for $a \rightarrow 0$ and $a \rightarrow \infty$ seems to be mandatory in ordinary quantum gravity. The reason was that the quantum formulation of wormholes has been shown to depend on the lapse function, \mathcal{N} [26,28]. The maximal analytical extension of minisuperspaces can be considered as the natural configuration space for quantization [26]. The boundary of the minisuperspace would then consist of all those configurations which are truly singular. Any regular configurations will be in its interior. Another reason to consider the above boundary conditions in a maximally extended minisuperspace is that it allow us to avoid boundary conditions at $a = 0$ to guarantee the self-adjointness of the Wheeler-DeWitt operator. This operator is hyperbolic and well posed boundary conditions can only be imposed on its characteristic surfaces and the one associated with $a = 0$ may not be of this type, like in the case of a conformally coupled scalar field. In such a case, it would be meaningless to require self-adjointness there (cf. ref. [26,28] for more details).

Within this framework wormhole solutions would form a Hilbert space. These ideas must then be extended to a case of locally supersymmetric minisuperspace with odd Grassmann (fermionic) field variables. In this case, not only one has to deal with different possible behaviours for \mathcal{N} but also with ψ_0^A . Then, it will be possible to determine explicitly the form of f, g, h, k in order that some or even an overlap of them could provide a wormhole wave function, including the ground state. In fact, this would mean that not only the bosonic amplitudes $A, B, ..$ would have to be considered for solutions but the fermionic pairs ought to be taken as well. Constructing an adequate Hilbert space from (3),(4) would lead us to a basis of wormhole states in such a singularity-free space (see [26]). Wormhole

wave functions could be interpreted in terms of overlaps between different states.

Another point which might be of some relevance is the following [28]. The evaluation of the path integral (or say, determining the boundary conditions for the Wheeler-DeWitt equation) for wormhole states in ordinary minisuperspace quantum cosmology requires the writing of an action adequate to asymptotic Euclidian space-time, through the inclusion of necessary boundary terms [15,26-28]. There may changes when fermions and supersymmetry come into play. A *different* action⁶ would then induces improved boundary conditions for the intervening fields as far a wormhole Hilbert space structure is concerned in a locally supersymmetric minisuperspace.

Summarizing, the issue concerning the existence of wormhole states in locally supersymmetric minisuperspace models was addressed in this work. Wormhole states are apparently absent in models obtained from the more general theory of N=1 supergravity with supermatter. As explained, the cause investigated here is that an appropriate choice of Lagrange multipliers and fermionic derivative makes a difference. From the former we get the simplest form of the supersymmetry and Hamiltonian constraints and their Dirac brackets in the pure case. This ensures no fermionic derivative ordering problems and that the solutions of the quantum constraints are consistent. Either from a *post-hoc* approach (trying to extend the obtained framework in the pure case) or from a direct dimensional-reduction we get consistent second order Wheeler-DeWitt type equations or corresponding solutions in the supermatter case. From an adequate use of criteria **1,2,3,4** above, we get a wormhole ground state. We also notice that the use of appropriate Lagrange multipliers also requires a specific fermionic ordering results in order to obtain a consistency set of Wheeler-DeWitt equations or respective solutions. The search for wormhole solutions could also be addressed from another point of view [28,30]. One has to invest supersymmetric quantum wormholes with a Hilbert space structure, associated with a maximal analytical extension of the corresponding minisuperspace. A basis of wormhole states might then be obtained from the many possible solutions of the supersymmetry constraints equations.

⁶ The canonical form of action of pure N=1 supergravity present in the literature [5] (which includes boundary terms) is not invariant under supersymmetry transformations. Only recently a fully invariant action but restricted to Bianchi class A models was presented [14].

Finally, I would like to quote the following words from C. Dickens book, “A Tale of Two Cities”:

It was the best of times, it was the worst of times; it was the age of wisdom, it was the age of foolishness; it was the epoch of belief, it was the epoch of incredulity; it was the season of Light, it was the season of Darkness; it was the spring of hope, it was the winter of despair; we had everything before us, we had nothing before us...

In my own opinion, it closely describes most of the path followed by some of us and which still remains ahead in the subject of supersymmetric quantum gravity/cosmology. Indeed, much more remains to be done in order to properly accomodate all basic results and avoid any paradoxical situations.

Note added

After completion of this work and before send it to the publishers, the author received a paper [43] by A. Csordàs and R. Graham. There, the problem of a cosmological constant in supersymmetric minisuperspaces from N=1 supergravity was dealt with and a solution proportional to exponential of the Chern-Simons functional was found.

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